Probability Based Scaffolding System with Fading

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Abstract. We propose a scaffolding system that provides adaptive hints using a probabilistic model, i.e., item response theory (IRT). First, we propose an IRT for dynamic assessment, whereby learners are tested under dynamic conditions of providing a series of graded hints. We then propose a scaffolding system that presents adaptive hints to a learner according to the estimated ability of IRT from the learner response data. The system provides hints so that the learner's correct response probability is 0.5. It decreases the number of hints (amount of support) automatically as a fading function according to the learner's growth capability. We conducted some experiments with students. The results demonstrate that the proposed system is effective.

Keywords: Learning science · Constructivism · Scaffolding · Dynamic assessment · Cognitive apprenticeship · Item response theory

1 Introduction

The leading metaphor of human learning has recently been transferred from instructionism to social constructivism [1],[2] in an education society. Vygotsky [1] introduced the Zone of Proximal Development (ZPD) with problem solving, by which a learner cannot solve difficulties alone, but can with an expert's help, thereby promoting learner development. Bruner (1978), like Vygotsky, emphasized the social nature of learning, reporting that other people should help a child develop skills through the process of "scaffolding" [3]. He defined scaffolding as steps taken to reduce the degrees of freedom in carrying out some task so that children can concentrate on difficult skills. The term "scaffolding" first appeared in the literature when Wood et al. described how tutors interacted with preschoolers to help them solve a block reconstruction problem [4]. Scaffolding situations were those in which learners obtained assistance or support to perform tasks beyond their own reach if pursued independently when unassisted. Stone (1998) emphasized the dynamic characteristics of the scaffolding process, which is dependent on cycles of assessment and adaptive support [5].

Brown and Ferrara [6] and Collins et al. [7] worked on a new method of assessment, called "dynamic assessment," by which a cascading sequence of hints was provided to enable dynamic assessment of how much support learners needed

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to complete various benchmark tasks. Subsequently, scaffolding was incorporated in cognitive apprenticeship theory [8] and has played important roles in several learning theories. Collins et al. [8] introduced "fading" to scaffolding, meaning that once learners accomplish a target skill, the teacher reduces (or fades) learner participation, providing only limited hints, refinements, and feedback to learners, who practice successive approximation of smooth executions of the whole skill. Pea (1993) claimed that scaffolding with fading is an intrinsic component that enabled what he called "distributed intelligence" [9].

Recently, a great deal of interest has arisen in the use of software tools to scaffold learners in complex tasks (e.g., [10]–[16]). However, these studies have been degraded in their usefulness because of three main problems.

1. No previous study has defined what common abilities in all tasks should be developed by scaffolding or dynamic methods of assessment.
2. Previous systems have been unable to predict learner performance with scaffolding based on their estimated abilities.
3. No previous study has used a reasonable strategy of how to scaffold learners. The strategies must provide appropriate support to increase learners' abilities.

Pea (2004) pointed out that many software features in the current scaffolding systems did not have a fading function [17]. The scaffolding system would necessarily derive the fading function if we were able to solve the three problems in the previous studies. The first problem is how to clarify the abilities developed by scaffolding. It is difficult to define the abilities directly because scaffolding does not directly transfer knowledge to learners but instead develops common abilities through solving tasks. However, in the test theory area, the representation of common abilities for all of tasks is known to be a latent variable model in item response theory (IRT) [18]. The probability of a correct response to a test item is modeled in IRT as a mathematical function of an individual latent ability that represents the common ability for all tasks. Our main idea is using this IRT to provide optimal help for scaffolding learners. We first propose an IRT model for dynamic assessment, by which learners are tested when dynamic conditions of providing a series of graded hints and estimate the model parameters from the obtained data. Next, we describe a scaffolding system that predicts the learner's performance with hints based on his/her estimated ability and presents adaptive hints to him/her. The system provides hints so that the learner's correct probability is 0.5. We assume an optimal correct response probability of 0.5 for scaffolding that can increase learners' abilities when the difficulty of tasks is slightly beyond the learners' abilities.

As a result, it automatically decreases the number of hints as a fading function, according to the learner's increasing ability. We conducted some actual experiments to demonstrate the effectiveness of the proposed scaffolding system by changing the predictive correct response probability. Results reveal that the adaptive hint function is the most effective in learning when we determine 0.5 to be the correct response probability. Therefore, over-assistance and lack of help hinder rather than support a learner's development.

2 Item Response Theory for Dynamic Assessment

2.1 Dynamic Assessment

The scaffolding process requires dynamic assessment to predict learner performance after a teacher's help is presented to them, as explained previously. Collins et al. compared the performance of children's responses to IQ test items under two conditions [8]. The first was "static assessment," which involved children trying to solve problems under conventional test conditions where they did not receive any help or guidance. The same children were also tested on the same items under dynamic conditions of providing a series of graded hints. The results demonstrated that dynamic assessment provided a stronger basis for predicting learning outcomes than static measures did. The most important result was that the greatest learning gain tended to be achieved by children who only needed minimal levels of guidance. The magnitude of the 'gap' between assisted and unassisted performance indicated by the amount of help needed was therefore prognostic of individual differences in learning outcomes. Assessing how much help a learner needed to succeed provided more decisive information about readiness for learning than determining how often they failed on the same, untutored tasks. Consequently, dynamic assessment integrated the assessment of learners' prior knowledge with the task of helping them to learn [12].

The problem with previous studies was that the number of hints needed was not a reliable measure of dynamic assessment because it strongly depended on the task difficulty. In addition, earlier studies did not clarify which ability should be developed by scaffolding or how to estimate it. In the next subsection, we propose an IRT model for dynamic assessment to resolve these problems.

2.2 Data from Dynamic Assessment System

We developed a dynamic assessment system to obtain learners' response data from tasks using a series of graded hints to apply IRT to dynamic assessment data.

We consider a series of graded hints $\{k\}$,($k = 1, 2, \ldots, K-1$) for task j. For that series, $k = 0$ when the task is presented without hints. First, the dynamic assessment system in a computer presents task j without hints to learner i.

If the learner responds incorrectly, then the system presents hint $k = 1$, or else the system stores the learner's response and presents the next task, $j + 1$. If the learner responds incorrectly to task j with hint $k = 1$, then the system presents hint $k = 2$; alternatively, the system stores the learner's response and presents the next task, $j + 2$. Consequently, the system presents hints from $k = 1$ to $k = K-1$ until the learner answers correctly. This procedure is repeated until $j = n$. Taking this procedure for N learners, we obtain dynamic assessment data

$$X = \{x_{ijk}\}, (i = 1, \cdots, N, j = 1, \cdots, n, k = 0, \cdots K), \tag{1}$$

where

$$x_{ijk} = \begin{cases} 1 : \text{learner } i \text{ answers correctly to task } j \text{ when hint } k \text{ is presented} \\ 0 : \text{else other,} \end{cases}$$

and x_{ijK} indicates the response data when learner i cannot answer correctly with hint $K-1$.

2.3 Item Response Theory for Dynamic Assessment

Item response theory [18], which is a recent test theory based on mathematical models, is widely being used in areas such as human-resource assessments, entrance exams, and certification tests with the widespread use of computer testing. It has three main benefits:

1. The learners' responses to different items can be assessed on the same scale.
2. It predicts the individual probability of correct answers from past response data.

We propose application of item response theory to data X obtained in dynamic assessment, where the problems with traditional dynamic assessment methods are solvable as a result of these three benefits. The probability, $p(u_j = k|\theta_i)$, that learner i will respond correctly task j after the k-th hint is assumed by the following graded response model [19]

$$p(u_j = k|\theta_i) = \frac{1}{1 + exp(-a_j\theta_i + b_{j_{(k-1)}})} - \frac{1}{1 + exp(-a_j\theta_i + b_{j_k})}, \tag{2}$$

where a_j stands for a discrimination parameter expressing the discriminatory power for learners' abilities of task j, b_{jk} is a difficulty parameter expressing the degree of difficulty of task j after the k-th hint is presented, and θ_i is an ability parameter expressing the ability of learner i. In addition, $p(x_j = 0|\theta_i) = 1$ and

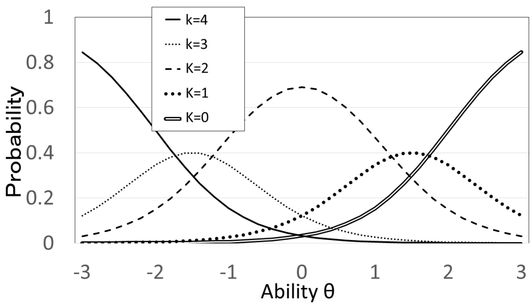

Fig. 1. Graded response model for hints

$p(x_j = K|\theta_i) = 0$. Here, we simply assume a unidimensional ability variable. Figure 1 depicts an example of item response function (2) for a task with four hints. The horizontal axis plots learners' abilities. The vertical axis plots the probability, $p(u_j = k|\theta_i)$, that learner i will correctly answer task j after k-th hint is presented.

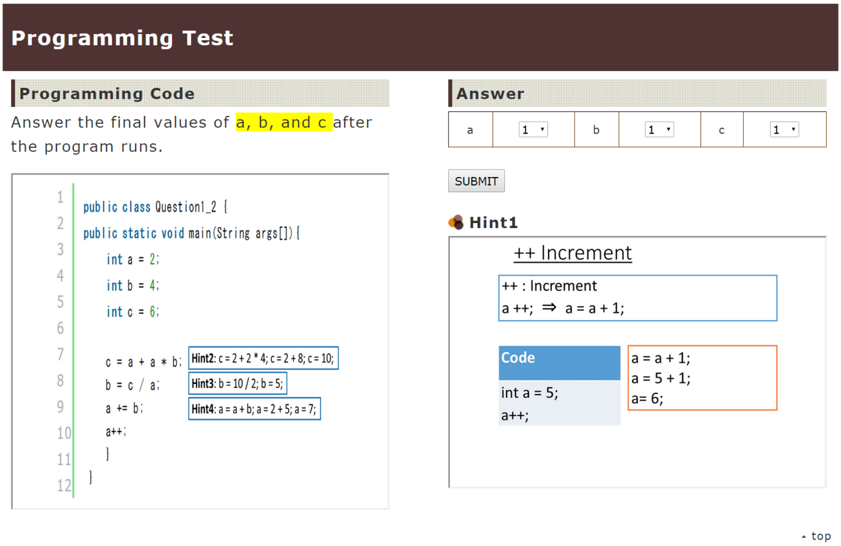

Fig. 2. Example of adaptive hints

2.4 Dynamic Assessment for Programming Trace Problems

We applied the proposed IRT to assess computer programming trace problems dynamically. We used the tasks to find the final numerical value of the target variable in the programs. We used seven tasks with four hints. The first hint presented required prior knowledge to solve the task, followed by successive hints with visualized trace results from the top of the program one after another.

We obtained response data X from 156 examinees using the dynamic assessment system. The examinees were first-year technical college students who had begun to study programming.

2.5 Estimated Parameters

We estimated the parameters of the graded response model in Eq. (2) using data X obtained in the previous section. We used the Newton–Raphson method to maximize the Bayesian posterior with a convergence criterion of 0.001. Table 1 presents the correct answer rates (CAR) for the tasks without hints, and shows the estimated parameters of a_j and b_{jk} for each task and associated hints.

Almost all tasks were slightly difficult from the CAR because all correct answer rates were less than 0.51. It is apparent from a_j that tasks 3–7 greatly discriminated learners' abilities but tasks 1 and 2 had poor discrimination. The estimated parameters, b_{jk}, for each hint were ordered according to the order in which the hints were presented because the hints were presented cumulatively. In the table, NA means that no learners answered correctly when a hint was presented. Therefore, there were only three available hints in task 7.

Table 1. Estimated parameters for each task and associated hints

	CAR	a_j	b_{j4}	b_{j3}	b_{j2}	b_{j1}	b_{j0}
Task 1	0.27	0.79	-2.59	-1.05	-0.54	0.23	0.99
Task 2	0.07	0.45	-1.62	-0.16	0.65	1.13	2.54
Task 3	0.26	2.03	-0.79	-0.25	0.33	0.77	1.06
Task 4	0.13	1.08	-1.04	-0.66	0.68	1.10	1.94
Task 5	0.37	1.02	-1.34	-0.52	-0.25	0.13	0.54
Task 6	0.37	1.15	-0.99	-0.66	-0.35	-0.20	0.52
Task 7	0.51	1.09	NA	-0.76	-0.57	-0.28	-0.04

We then compared the reliabilities of the ability estimators with the numbers of hints that were used in previous studies [8] of dynamic assessment. We calculated the correlation coefficients between the estimated abilities using data for tasks 1–4 and those using data for tasks 4–7. The results revealed a high correlation coefficient value of 0.862. We similarly calculated correlation coefficients between the average number of hints needed for tasks 1–4 and those for tasks 4–7. We obtained a comparatively low value of 0.662. The main reason the number of necessary hints was less reliable is that the variance of the numbers of used hints tended to be small because only a few hints were needed for learners; then the magnitude of estimation error tended to be large. In contrast, the proposed estimated ability for dynamic assessment was a more reliable measure by minimizing the effects of heterogeneous or aberrant responses that might have affected poor accuracy in the estimates. Consequently, the proposed method improves the reliability of dynamic assessment.

3 Probability Based Scaffolding System with Fading

Our main interest in this study was to clarify the mechanism for effective scaffolding. The main difficulty with scaffolding is that over-assistance or lack of help interrupts effective learning. The problem is how to optimize the magnitude of help using dynamic assessment. Here, we introduce a method of presenting adaptive hints to control learners' predictive correct response probabilities in tasks. Here, we assume that some optimal correct response probability to increase learners' abilities that is achieved by scaffolding when the difficulty of tasks is slightly beyond the learners' abilities. The most important problem is to ascertain how great the optimal correct response probability is. We assume that the optimal probability is 0.5 in this study because this is the borderline level of help to enable the learners to solve the task.

According to this idea, we developed a scaffolding system to solve the programming trace problem. Fig. 2 depicts an example of the system by which all of four hints are presented. First, the system presents the first task without hints. In Fig. 2, the task is presented on the left of screen. If a learner answers correctly, then the system estimates the learner ability using the learner response data; then the system presents the next task. Here, the initial value of θ_i is zero, which is the average of θ_i. If the learner answers incorrectly, then the system searches the hint, with which the learner predictive correct answer probability is nearest to 0.5 from the hints' database. The learner's predictive correct answer probability, $p(u_j = k|\theta_i)$, is estimated using the learner's estimated ability θ_i and hint parameters a_j, b_{jk} stored in the database. Then, the system presents the selected hint to the learner. On the right of screen in Fig. 2, Hint 1 is presented to explain "increment :++" in the program. The system re-estimates the learner's ability and presents the optimal hint from the remaining hints in the database if the learner answers the task incorrectly with the hint. This procedure is repeated until the learner answers correctly or until there are no remaining hints in the database. On the left of screen in Fig. 2, Hint 2, Hint 3, and Hint 4 are presented sequentially. The system presents the next task if the learner answers correctly.

This algorithm was inspired by adaptive testing that presented optimal items for measuring learners' abilities.

4 Evaluation Experiment

This section explains how we evaluated the proposed scaffolding system using actual data. The participants in these experiments were 93 first-year university students of the faculty of engineering who had begun to study programming.

4.1 Method

The participants were divided into six groups (A–F) for different experiments.

A) The system presented hints so that the learner's predictive correct answer probability was close to 0.8.
B) The system presented hints so that the learner's predictive correct answer probability was close to 0.65.
C) The system presented hints so that the learner's predictive correct answer probability was close to 0.5 (proposed method).
D) The system presented no hints. (the learner's predictive correct answer probability was 0.1–0.5). The system presented the correct answer if the learner answered incorrectly once.
E) The system presents the graded hints sequentially in the same way as the method explained in section 2.2. The system presents the next hint if the learner answers incorrectly. This procedure was repeated until the learner answered correctly.

F) The system presents the correct answer, and provides an explanation for this if the learner answers incorrectly once. The explanation included the contents for all the hints.

We developed these six versions of the system. The experiments were conducted according to five steps:

1. The examinees took a pre-test to assess their prior knowledge using the system. The pre-test consisted of programming trace problems asking for the final values of variables after the program began working. The examinees had to solve the problems without hints by themselves.
2. The system presented basic knowledge related to programming trace problems to the examinees after the pre-test had taken place.
3. The system started the scaffolding module corresponding to each group (A-F) after previous learning had taken place.
4. The examinees took a post-test after learning with the scaffolding system. The post-test consisted of new problems combined with the previously learned programming grammars. The examinees had to solve the problems by themselves without hints.
5. After a week, the examinees took a memory holding test that consisted of similar items to those in the post-test.

4.2 Results

Evaluation of Basic Functions. This section explains our evaluation of the basic functions of the proposed system. We first tested and confirmed that the system presented adaptive hints so that the learners' correct answer rates were close to 0.5. Figure 3 depicts the average correct answer rates over all examinees for tasks when hints were presented. The error bar shows the standard error. Figure 3 also indicates that the system controlled learners' correct answer probabilities are around 0.5 by presenting adaptive hints to various levels of learners. This evidence of control demonstrates that the function of adaptive hints functioned precisely because the correct answer rates without hints were between 0.1 and 0.37 except for task 7. The average correct answer rate tended to be higher than 0.6 for task 7, whose correct answer rate was beyond 0.5. We tested and confirmed that the system increased the learners' abilities. Figure 4 depicts the average estimated abilities for tasks when learning with the system. The average ability increases monotonically from 0.1 in the figure when learners proceed with learning until task 6; it converges to around 0.4. This result demonstrates the effectiveness of the proposed system for learner development.

We subsequently confirmed the fading function of the system. Figure 5 depicts the transition in the average number of hints presented to learners in the system. The number of presented hints does not decrease monotonically because the characteristics of hints differ for tasks. However, the average number of presented hints decreases dynamically after learning task 4. The system gradually decreases the amount of help according to increases in learner's ability. This is the fading function that is expected to enhance learners' autonomous learning and their self-reliance in solving tasks.

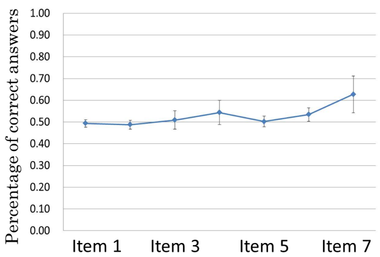

Fig. 3. Average correct answer rate

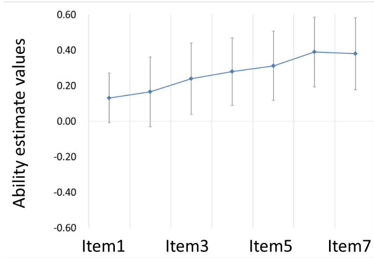

Fig. 4. Transition in estimated abilities

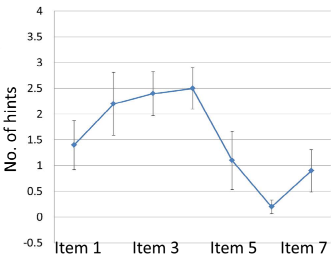

Fig. 5. Average number of presented hints

Evaluation of Scaffolding. This section presents a comparison of the performance of pre-test and post-test examinees groups from A to F that were used to evaluate the proposed system. The test results are presented in Table 2, which lists the number of examinees allocated to each group, the average score obtained from pre-tests, the average score from post-tests, the average score from memory-holding tests, and the average learning time using the system. The values in parentheses in the table represent standard errors. The χ^2 test with a significance level of 5% indicates that the results from the pre-test are equivalent to those of the other groups. Therefore, no differences were found in the groups before the experiment. In addition, the average pre-test scores were extremely low because the examinees were beginners at programming.

We assessed differences between groups using one-way analysis of variance (ANOVA) in the results from post-tests; then we used the Tukey–Kramer method for the detected differences. The proposed scaffolding method outperformed the others, from Table 2, with a significance level of 5% despite the short average learning times. Conversely, Group F, which provided the answers and their explanations, exhibited the worst performance, although the explanation included the content for all hints. This method provided less opportunity for learners for deep consideration of problems because the average learning time was the shortest. In contrast, group D, with no hints, provided the second-best performance. The average learning time for group D was longer than that for group F. Presenting answers only after learners' incorrect answers might induce deep thinking from this to solve problems. Moreover, this result suggests that over-instruction is ineffective for attaining learner development.

The system presented herein hints in groups A and B so that learners' predictive correct answer probability was near 0.8 for the former and 0.65 for the latter. In these cases, the system tended to present more help (content of hints) than that for group C. The average scores for groups A and B were less than that of group D, although the averages of learning times for groups A and B were longer than those for the others. This result shows that setting correct answer probabilities by scaffolding strongly affects learning effectiveness. We conducted a conventional dynamic assessment procedure for group E. The average score for group E was almost identical to those for groups A and B from the results. Actually, the effectiveness of the conventional method was the same as that of the other methods with slight over-assistance. The proposed method, group C, provided the best average score in the results for the memory holding test. In contrast, the average score for group F was the worst. The average scores for the other groups were almost identical. These results indicate that the proposed scaffolding method with a correct answer probability of 0.5 was superior.

We also administered two questionnaires to the examinees: 1) Did you think that you achieved the correct answers to the tasks by yourself? and 2) Did you have confidence in solving similar tasks by yourself? The examinees answered them by responding on a five-point Likert scale:
1. Strongly disagree, 2. Weakly disagree, 3. I am not sure, 4. Weakly agree, and 5. Strongly agree.

The results, presented in Table 3, indicate that the proposed method has the best scores. Therefore, the proposed method enabled learners to think that they could solve tasks independently.

Table 2. Results from pre- and post-tests (Tukey–Kramer method and significant difference from group C: *5 %, **1%)

Group	A	B	C	D	E	F
No. of subjects	14	16	18	15	12	18
Pre-test score	1.14 (1.59)	1.69 (2.44)	1.78 (2.44)	1.33 (1.89)	2.17 (1.40)	2.72 (2.23)
Post-test score	35.4** (2.94)	34.8** (2.13)	40.0 (3.15)	36.5* (2.22)	34.8** (2.44)	30.9** (4.92)
Memory holding test	20.8 (2.73)	20.8 (2.27)	23.0 (2.18)	20.6 (1.81)	20.8 (1.81)	18.3 (5.41)
Learning time (min)	69 (26)	78 (28)	71 (22)	67 (15)	72 (24)	64 (24)

Table 3. Results from questionnaires (Tukey–Kramer method and significant difference: *5%)

Group	A	B	C	D	E	F
Questionnaire 1	2.57 (0.979)	2.16 (0.601)	3.00*(0.882)	2.06(0.680)	2.31(0.583)	2.00*(0.577)
Questionnaire 2	3.79 (0.340)	3.81 (0.674)	4.05 (0.726)	3.87 (0.705)	3.75 (1.01)	3.67 (1.00)

5 Conclusions

This article proposed a scaffolding system that provided adaptive hints using a probabilistic model, i.e., item response theory (IRT). We first proposed IRT for dynamic assessment in which learners were tested under dynamic conditions of providing a series of graded hints. We then explained a scaffolding system we had developed that presented adaptive hints using the estimated ability using IRT from learner's response data. The system provided hints so that learner's correct response probability was 0.5. We conducted some experiments with the students and obtained four results: 1) The scaffolding system enhanced learner development to increase the learner ability. 2) The system achieved scaffolding with fading. 3) Neither over-instruction nor lack of instruction was effective for learner development. 4) Scaffolding so that learners' correct answers were 0.5 provided superior results for learner development.

We have three plans for future work: A) We intend to increase the number of hints because the proposed system will become more effective as the number of hints increases. B) We intend to expand IRT to one with multidimensional abilities or to Bayesian network because the unidimensional ability model has limitations that are too strict for actual scaffolding processes. C) We did not consider unique features in which the estimated ability was dynamically increased in the system design. Discarding response data from earlier presented tasks might improve the accuracy of estimating a learner's current ability.

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